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Controller Order Reduction using Optimal HANKEL NORM Approximation Optimized with PSO

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Abstract—*The design and implementation of full-order controllers often requires advanced hardware and high computational processing effort mainly for problems that involves large models. The advanced controller designs methods like LQG/LTR or H-infinity based synthesis methods leads to controllers of order comparable to the plant and sometimes unstable controller whereas lower order controller should be sought out to keep the closed-loop stability and system performance within acceptable limit. To avoid this, it is recommended to use reduced order controllers. Here we take an unstable controller and reduce the controller using optimal hankel norm approximation and then optimized the results using Particle Swarm Optimization (PSO).*

Keywords: *Controller reduction, Optimal Hankel Norm Approximation, PSO, Singular value decomposition.*

1. INTRODUCTION

Minimization of H-infinity norm is one of the criteria in controller reduction for active vibrations, which consists of peak frequency response attenuation of the controlled system, while maintaining stability. The development of the optimal hankel norm approximation [1, 2] and the balanced truncation [3] changed the perception of model reduction significantly. By applying these two techniques we can ensure that the reduced order controller will almost have the same characteristics as the original one and we can also ensure stability of the system.Admajan et al. [4 introduced a closed form optimal [4] to model reductions with respect to hankel norm criterion for SISO systems. This technique was first applied on MIMO systems by Kung [2] who also mentioned the relevance of model reduction.To derive explicit algorithms and simple expressions for the Hankel norm approximation of a high dimensional discrete-time stable scalar system by a reduced model of any low order,the structure of linear dynamical systems with finite dimension is exploited in [6]. Bay et al [7] solved the case of continuous-time scalar, in addition to that Glover [1] investigated characterization of all optimal Hankel norm approximations that minimize the Hankel norm for multivariable linear systems and derived the frequency response error bound

In this paper a controller reduction algorithm with frequency weightings based on optimal Hankel norm approximation (OHNA) technique is discussed.

2. PROBLEM FORMULATION

When reduced order controller replaces the full order controllerwe emphasize on the reduced order controller that it approximates the closed loop system and does not violates any closed loop objectives. Taking note of these objectives, we also introduced a frequency weighting selection scheme is considered in conjunction with the controller reduction algorithm via OHNA algorithm. The stability or performance using weights is scrutinized in [12], [18]. To guarantee the closed-loop stability the frequency weighted balancing related approaches can be applied to controller reduction problems by preserving the H-infinity performance bound which were achieved by using the original controller.

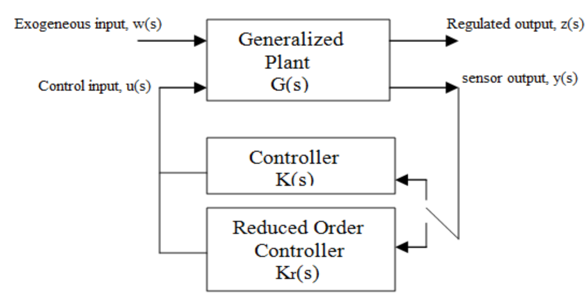

Fig. 1: Generalizedplantandcontroller interconnection

Consider a generalized two port plant and controller configuration as shown in Fig. 1 with transfer function matrix G(s), high order controller K(s), reduced order controller $K_r(s)$, external input w, controlled output z, control input u and measured output y. The closed loop transfer function $T_{zw}(s)$ which is the T.F. matrix from w(s) to z(s) with the controller K(s) connected to the plant G(s) is given by lower linear fractional transformation (LFT) of G(s) and K(s) [15], [19]. The plant G, full order controller K and reduced order controller K_r are given by

$$G = \begin{matrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{matrix} = \begin{matrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{matrix} \quad (1)$$

$$K = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \ (2)$$

$$K_r = \begin{bmatrix} A_r & B_r \\ C_r & D_r \end{bmatrix} \ (3)$$

Then the Closed-loop transfer function with full order controller K(s) and reduced order controller $K_r(s)$ are given by

$$T_{zw\,(s)} = F_1(G, K) = G_{11} + G_{12}K(1 - G_{22}K)^{-1}G21 \ (4)$$

$$T_{zw\,(s)} = F_1(G, K_r) = G_{11} + G_{12}K_r(1 - G_{22}K_r)^{-1}G21 \ (5)$$

Minimizing $\|T_{zw}(s) - T_{zw}(s)\|$ results in the optimal solution [15] which sometimes may not be achieved, so we compute a stabilizing reduced order controller $K_r(s)$ such that $\|T_{zw}(s) - Tzw(s)$< γ where γ is a positive constant.

3. UNSTABLECONTROLLER DECOMPOSITION ALGORITHM

The system decomposition algorithm [20] is adopted for decomposition of unstable controllers. It contains two stages of transformations. In first stage the block form of real Schur transformation is used whereas in second stage of transformation, the generalized Lyapunov equation has been solved for obtaining decomposed stable and unstable subsystems. The decomposition algorithm consists of following steps:

Step 1: The unstable controller K is transformed into block diagonal upper Schur form using an unitary matrix U. If x denotes the states of unstable controller then the first stage transformation matrix U (the unitary matrix) and the transformed system states x_t may be related as $x = U x_t$ Thus, the first stage transformed system becomes,

$$K_t = \left[\begin{array}{c|c} U^T A_k U & U^T B_k \\ \hline C_k U & D_k \end{array}\right] = \left[\begin{array}{c|c} A_t & B_t \\ \hline C_t & D \end{array}\right] =$$

$$\begin{array}{c} \overset{m}{\longleftrightarrow} \ \overset{n-m}{\longleftrightarrow} \\ \left[\begin{array}{cc|c} A_{t11} & A_{t12} & B_{t1} \\ 0 & A_{t22} & B_{t2} \\ \hline C_{t1} & C_{t2} & D \end{array}\right] \end{array} \begin{array}{l} \updownarrow m \\ \updownarrow n-m \\ \\ \end{array} \quad (6)$$

Where n denotes order of the system, m denotes the number of stable eigenvalues and n-m denotes the number of unstable Eigen values.

Step 2: The transformed system of step (1), contains a coupling term A_{t12}. To bring transformed system into completely decoupled form, the general form of Lyapunov equation is used.

$$A_{t11}S - SAt_{22} + A_{t12} = 0 \ (7)$$

The value of *S* is obtained and second stage of transformation is carried out using $x_t = WX$ where *X* is the final stage transformed state and *W* is the final stage transformation matrix. The second stage transformation matrix *W* is given as

$$W = \begin{bmatrix} I_m & S \\ 0 & I_{n-m} \end{bmatrix} \quad (8)$$

The important property of *W* is that W^{-1}can be obtained simply by replacing *S* with –*S*. i.e.

$$W^{-1} = \begin{bmatrix} I_m & -S \\ 0 & I_{n-m} \end{bmatrix} \quad (9)$$

Using *W*, the completely decoupled system (K_d) is obtained as,

$$K_d = \begin{bmatrix} W - 1 \ A_t W & W^{-1}B_t \\ C_t W & D \end{bmatrix}$$

$$K_d = \begin{bmatrix} A_{k11} & 0 & B_1 \\ 0 & A_{k22} & B_2 \\ C_1 & C_2 & D \end{bmatrix} \quad (10)$$

This transformed model may be decomposed into stable and unstable as

$$K_d = \begin{bmatrix} A_{k11} & B_{k1} \\ C_{k1} & D \end{bmatrix} + \begin{bmatrix} A_{k22} & B_{k2} \\ C_{k2} & 0 \end{bmatrix} \ (11)$$

= K_S (stable part) + K_U (unstable part)

4. FREQUENCY WEIGHTED CONTROLLER REDUCTION BY OPTIMAL HANKEL NORM APPROXIMATION

Step 1: Choice of weightings

Choose the weighting T.Fs. according to one of the following criterion [18]:

(i) To enforce closed loop stability, single-sided weightings of the following form can be chosen

Either $W_i = I \ and \ W_o = (I + GK)^{-1}G$ (12)

Or,$W_i = (I = GK)^{-1} \ and \ W_o = (I + GK)^{-1}G$ (13)

Step 2: Determination of Gramians

Let the input and output weightings with the following minimal realization [19]

$$W_i = \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} \ and \ W_o \begin{bmatrix} A_o & B_o \\ C_o & D_o \end{bmatrix} \ (14)$$

Let the Gramians are of the following form

$$P_{ai} = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} \; and \; Q_{ao} = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix} \quad (15)$$

Step 3: Determination of Hankel Singular values

Hankel Singular values (HSV) of the system are obtained by simultaneously diagonalization of weighted Gramians P and Q.

$$TP_eT^T = (T^{-1})^T Q_e T^{-1} = \Sigma \quad (16)$$

$$= diag(\sigma_1 \sigma_2, \ldots \ldots \ldots, \sigma_k \sigma_{k+1}, \ldots \ldots \ldots, \sigma_m)$$

Then, Gramians [16] become

$$\Sigma = \begin{bmatrix} \hat{\Sigma} & \\ & \sigma_{r+1} I_l \end{bmatrix} \quad (17)$$

Step 4: Now for finding similarity transformation matrices S_L and S_R Let,

$$V_R = U_P \sqrt{\Sigma_P}, V_L = U_q \sqrt{\Sigma_q}, E = V_L' V_R \quad (18)$$

Singular value decomposition of E is obtained such that

$$E = U_E \Sigma_E V_E' \quad (19)$$

To compute similarity transformation matrices S_L and S_R for balanced realized model of the non-minimal system the elements of Σ_E is truncated beyond minimal order k of the controller as

$$\Sigma_{trunc} = \Sigma_E(1{:}k, 1{:}k) \quad (20)$$

Finally the transformation matrices are obtained as

$$S_L = V_L U_E (\Sigma_{trunc})^{-1/2}$$

$$S_R = V_R V_E (\Sigma_{trunc})^{-1/2} \quad (21)$$

Step7: Obtain HNA model

The minimal balanced realized model is partitioned according to the partition of the Gramians as

$$A_{bal} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, B_{bal} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, C_{bal} = [C_1 \quad C_2]$$

Step 8: The overall reduced order controller is obtained by adding the reduced model computed in step 7 and the decomposed unstable part of original controller as

$$K_{ohna} = K_{sr\_hna} + K_U \quad (22)$$

5. PARTICLE SWARM OPTIMIZATION

Swarming strategies in bird flocking and fish schooling are used in PSO introduced by Eberhart and Kennedy. Each Particle in the swarm is represented by the following characteristics:-

1. The current position of the particle
2. The current velocity of the particle

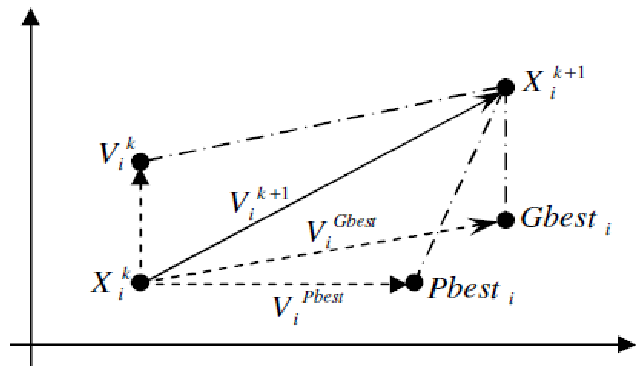

Fig. 2: Concept of modification of a searching point by PSO

6. ILLUSTRATIVE EXAMPLE

Consider the generalized 3rd order stable plant [19] with following parameters for H-infinitybased controller reduction

$$G = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 4 & 0 & 0 & 1 \\ 0 & -2 & 0 & 1 & 0 & 1 \\ 0 & 0 & -3 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

For the above plant Zhou [19] has discussed the 2 H norm based controller and its reduction but here ∞ H based full order suboptimal controller reduction has been considered. For the above 3rd order plant ∞ H based full order suboptimal controller K (s) at suboptimum value γ = 0.4493, is obtained as

$$K = \frac{-30738 * 10\text{^}11(s+3)(s+1)}{(s+6.838*10\text{^}11)(s+3.282)(s-0.1815)}$$

The above controller is unstable, for reduction of this controller we decompose it into stable part K_S and unstable part K_U, then by using algorithm which we discussed reduce the stable part

$$K_s = \frac{-3.0738 * 10\text{^}11(s+3.096)}{(s+3.282)(s+6.838*10\text{^}11)}$$

$$K_U = \frac{-0.48787}{(s-0.1815)}$$

Now the reduced controller is

$$K_{red} = \frac{-3.0738 * 10^{11}(s + 0.9038)}{(s + 6.6838 * 10^{11})(s - 0.1815)}$$

This reduced controller is optimized by PSO and obtained as

$$K_{new} = \frac{-3.6454 * 10^{11}(s + 0.8121)}{(s + 8.421 * 10^{11})(s - 0.1823)}$$

The step responses and singular value plots for closed-loop systems with original, reduced order controller and PSO optimized controllerare shown in Figurerespectively.

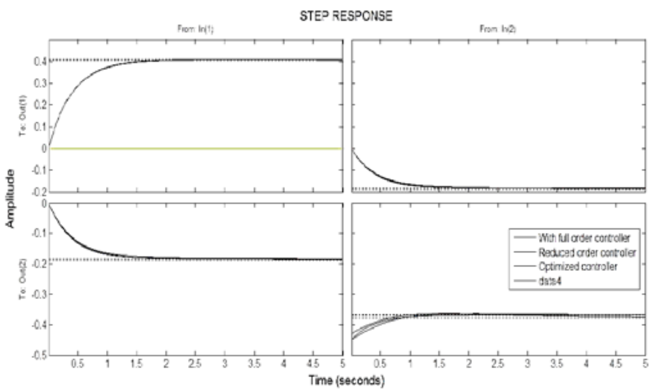

Fig. 3: Step response of different controller

7. CONCLSION

As we can see that the controller which is designed by optimal hankel norm approximation exhibits and approximates the closed loop performance of the controller which is original designed. The results after optimizing the results further improves.

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